# EXERCISE – III

## HINTS & SOLUTIONS

**Sol.1** $\frac{(1+i)x-2i}{3+i} + \frac{(2+3i)y-i}{3-i} = i$

(3 – i) [x +i (x – 2)] + [2y + i (1 – 3y)] (3 + i) = 1i

(3x + x – 2) + i [3x – 6 – x] +(6y – 1 + 3y) + i(2y + 3 – 9y) = 12i

(4x + 9y – 3) + i [2x – 7y – 3] = 10i

By comparing

x = 3, y = –1

**Sol.2** **(i)** Let $\sqrt{7+24i}$ = x + iy

$7 + 24i = x^2 - y^2 + 2ixy$

$x^2 + y^2 = 7$ ....(1)

xy = 12 ....(2)

$(x^2 + y^2)^2 = (x^2 - y^2) + 4x^2y^2$

= 42 + 576

= 625

$x^2 + y^2 = \pm 25$

$x^2 + y^2 = 2\ 5$ ...(3)

$x^2 - y^2 = -25$ (reject)

From (1) and (3)

$2x^2 = 32 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

$x = -4 \Rightarrow y = -3$

$\sqrt{7+24i} = \pm (4 + 3i)$

**(ii)** Let $\sqrt{4+3i}$ = x + iy

$4 + 3i = x^2 - y^2 + 2ixy$

$x^2 - y^2 = 4$

2xy = 3

$(x^2 + y^2)^2 = (x^2 - y^2) + (2xy)^2 = 16 + 2$

$x^2 + y^2 = 5$

$x^2 = \frac{3}{2} \Rightarrow x = \pm \frac{3}{\sqrt{2}}$

$x^2 = \frac{3}{\sqrt{2}} \Rightarrow y = \frac{1}{\sqrt{2}}$

$x = -\frac{3}{\sqrt{2}} \Rightarrow y = -\frac{1}{\sqrt{2}}$

$\sqrt{4+3i} = \pm \frac{1}{\sqrt{2}}(3 + i)$

**Sol.3** **(a)** $z = 1 + \cos\frac{18\pi}{25} + i\sin\frac{18\pi}{25}$

Let $\theta = \frac{18\pi}{25} = 1 + \cos\theta + i\sin\theta$

$= 2\cos^2\frac{\theta}{2} + 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$= 2\cos\frac{\theta}{2}\left[\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right]$

$= 2\ \text{cdos}\ \frac{\theta}{2}\ e^{i\frac{\theta}{2}}$

$|z| = 2\cos\frac{\theta}{2} = 2\cos\frac{3\pi}{25}$

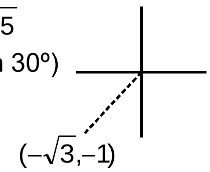
$\text{Arg}(z) = \frac{\theta}{2} = \frac{2\pi}{25} + 2n\pi \quad n \in I$

Principal Arg (z) = $\frac{9\pi}{25}$

**(b)** z=– 2 (cos 30º+i sin 30º)

$= -\sqrt{3} - i$

|z| = 2




$\text{Arg } z = -\pi + \frac{\pi}{6} = \frac{-5\pi}{6} + 2n\pi$

$n \in I$

Principal Arg (z) = $\frac{-5\pi}{6}$

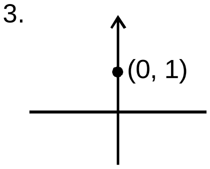
**Sol.4** **(a)** 1 < |z – 2i| < 3

|z – α| = 2

denotes the circle with centre α and β radius a so z denotes the concentric circle cuts (0, 2) and is between radius of 1 & 3.

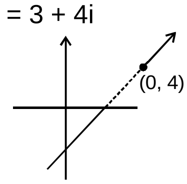
**(b)** Im(z) ≥ 1

A ray on positiv y-axis on or above 1




**(c)** Arg (z – a) = $\frac{\pi}{3}$ ' a = 3 + 4i

Arg((x – 3) + i (y – 4)) = $\frac{\pi}{3}$




$y - 4 = \sqrt{3}(x - 3)$

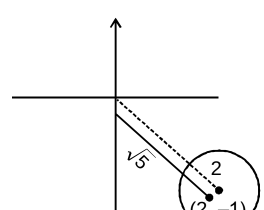
A ray emanating from the point (3 + 4i) directed away from the origin & havin the equation & having the equation

$y - 4 = \sqrt{3}(x - 3);\ x > 3$

**Sol.5** |z – 2 + i| ≤ 2

max. value = $2 + \sqrt{5}$

max. value = $\sqrt{5} - 2$



**Sol.6** $|z + 3| \le 3$

**(i)** max of $|z| = 3 + 3 = 6$

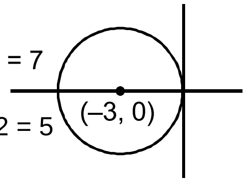
max. of $|z| = 0$

**(ii)** max. of $|z - 1| = 1 + 6 = 7$

max. of $|z - 1| = 0$

**(iii)** max. of $|z + 1| = 3 + 2 = 5$

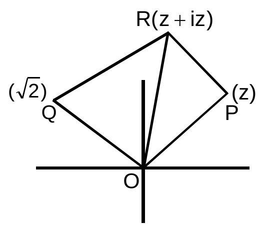
max. of $|z + 1| = 0$




**Sol.7** Area of $\Delta PQR = 200$

$$\frac{1}{2}|z| \times |z| = 200$$

$|z| = 20$




**Sol.8** $|z_1| = |z_2| = |z_2| = .... = |z_n| = 1$

**(i)** $|z_1| = 1$

$|z_1|^2 = 1$

$$z_1\bar{z}_1 = 1 \Rightarrow \bar{z}_1 = \frac{1}{z_1}$$

**(ii)** $$\left|\frac{1}{z_1} + \frac{1}{z_2} + \ldots + \frac{1}{z_n}\right|$$

$= |\bar{z}_1 + \bar{z}_2 + \ldots + \bar{z}_n|$

$= |\overline{z_1 + z_2 + \ldots + z_n}|$

$= |z_1 + z_2 + \ldots + z_n|$

$=$ LHS H.P.

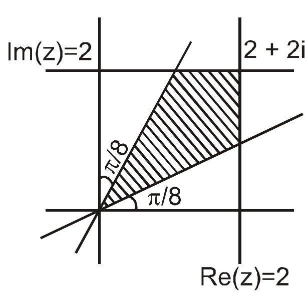
$\because$ $|z_1| = 1 \Rightarrow \left|\frac{1}{z_1}\right| = 1$

$|z_2| = 1 \Rightarrow \left|\frac{1}{z_2}\right| = 1$

$|z_n| = 1 \Rightarrow \left|\frac{1}{z_n}\right| = 1$

Hence that 2n points are the vertices of a regualr polygon.

**Sol.9** $\text{Re}(z) \le 2$, $\text{Im}(z) \le 2$, $\frac{\pi}{8} \le \arg(z) \le \frac{3\pi}{8}$




**Sol.10** **(i)** $(1 + \cos\theta + i\sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n$

$$= \left(2\cos^2\frac{2\theta}{2} + i2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)^n$$

$$+ \left(2\cos^2\frac{\theta}{2} - i2\cos\frac{\theta}{2}\sin\frac{\theta}{2}\right)^n$$

$$= 2^n\cos^n\frac{\theta}{2}\left[\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)^n + \left(\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}\right)^n\right]$$

$$= 2^n\cos^n\frac{\theta}{2}\left[\cos\frac{n\theta}{2} + i\sin\frac{n\theta}{2} + \cos\frac{n\theta}{2} - i\sin\frac{n\theta}{2}\right]$$

$$= 2^{n+1}\cos^n\frac{\theta}{2} = \text{RHS}$$

**(ii)** $(1 + i)^n + (1 - i)^n$

$$(\sqrt{2}\, e^{i\frac{\pi}{4}})^n + (\sqrt{2}\, e^{-i\frac{\pi}{4}})^n$$

$$= 2^{\frac{n}{2}}\left[e^{i\frac{n\pi}{4}} + e^{-i\frac{n\pi}{4}}\right]$$

$$= 2^{\frac{n}{2}+1}\cos\frac{n\pi}{4}$$

**Sol.11** $(z - 1)^4 = 16$

$(z - 1) = (16)^{1/4}$

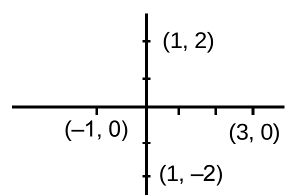
$$z - 1 = 2\left(\cos\frac{2m\pi}{4} + i\sin\frac{2m\pi}{4}\right)$$

where $m = 0, 1, 2, 3$

Roots will $z = -1, 3, 1 - 2i, 1 + 2i$

Sum = 4

centroid = (1, 0)




**Sol.12** **(i)** $$\left(\frac{1}{2} + \frac{\sqrt{-3}}{2}\right)^3 = \left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^3$$

$= -(\omega^2)^3 = -(\omega^3)^2 = -1$

**(ii)** $$\left(\frac{1}{2} + \frac{\sqrt{-3}}{2}\right)^{3/4} = \left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{3/4}$$

$$= \left[e^{i\left(2n\pi + \frac{\pi}{3}\right)}\right]^{3/4}$$

$= e^{i(6n+1)\frac{\pi}{4}}$ where $n = 0, 1, 2, 3$

Product = –1

**Sol.13** **I :** Arg $\left(\frac{z-8i}{z+6}\right) = \pm\,\frac{\pi}{2}$

I represent a circle with diameter ends are $(-6, 0)$ & $(0, 8)$

Hence equation of circle

$(x + 6)\,x + y\,(y - 3) = 0$

$x^2 + 6x + y^2 - 8y = 0$

**II :** Re $\left(\frac{z-8i}{z+6}\right) = 0$

Re $\left(\frac{x+i(y-8)}{(x+6)+iy} \times \frac{(x+6)-iy}{(x+6)-iy}\right) = 0$

Hence I and II represent same circle

$\frac{z-8i}{z+6} + \frac{\bar{z}+8i}{\bar{z}+6} = 0$

$(z - 8i)\,(\bar{z} + 6) + (z + 6)\,(\bar{z} + 8i) = 0$

$z\bar{z} + z\,(3 + 4i) + \bar{z}\,(3 - 4i) = 0$

It is also represent +ve same circle

$|z + 3 - 4i| = R$

$(x + 3)^2 + (y - 4)^2 = R^2$

$x^2 + 6x + y^2 - 8y + 9 + 16 = R^2$

$\Rightarrow R^2 = 25 \qquad \Rightarrow R = 5$

**Sol.14** $\sum_{r=1}^{n-1}(n-r)\alpha^r$

$= (n - 1)\,\alpha + (n - 2)\,\alpha^2 + (n - 3)\,\alpha^3 + \ldots + \{n - (n - 1)\alpha^{n-1}$

$= n\,[\alpha + \alpha^2 + \alpha^3 + \ldots + \alpha^{n-1}] - [\alpha + 2\alpha^2 + 3\alpha^3 + \ldots + (n-1)\alpha^{n-1}]$

$S = -n - S_1 \qquad \ldots(1)$

$S_1 = \alpha + 2\alpha^2 + 3\alpha^3 + \ldots + (n - 1)\,\alpha^{n-1}$

$\alpha S_1 = \alpha^2 + 2\alpha^3 + \ldots + (n - 2)\,\alpha^{n-1} + (n - 1)\,\alpha^n$

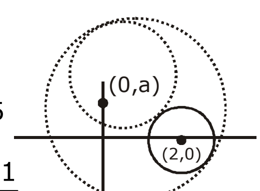
$S_1\,(1 - \alpha) = \alpha + \alpha^2 + \alpha^3 + \ldots + \alpha^{n-1} - (n - 1)\alpha^n$

$S_1(1 - \alpha) = -1 - n\alpha^n + \alpha^n$

$S_1 = \frac{-n\alpha^n}{1-\alpha}$

$S = -n\,\frac{+n\alpha^n}{1-\alpha} = -n + \frac{n}{1-\alpha} = \frac{-n+n\alpha+n}{1-\alpha} = \frac{n\alpha}{1-\alpha}$

**Sol.15** For two circles




$|r_1 - r_2| < c_1c_2 < r_1 + r_2$

$|a + 3| < \sqrt{a^2+4} < a+5$

$a < -\frac{5}{6} \qquad a > -\frac{21}{10}$

$a \in \left(-\frac{21}{10}, \frac{-5}{6}\right]$

**Sol.16** $x^3 - 3x^2 + 3x + 7 = 0$

$(x - 1)^3 = -3$

$\frac{x-1}{-2} = (1)^{1/3} = 1, \omega, \omega^2$

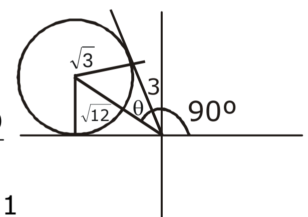
$\alpha = -1,\ \beta = 1 - 2\omega,\ \gamma = 1 - 2\omega^2$

$\frac{\alpha-1}{\beta-1} + \frac{\beta-1}{\gamma-1} + \frac{\gamma-1}{\alpha-1}$

$= \frac{-2}{-2\omega} + \left(\frac{-2\omega}{-2\omega^2}\right) + \left(\frac{-2\omega}{-2}\right)$

$= \frac{1}{\omega} + \frac{1}{\omega} = \frac{1}{\omega} = \frac{3}{\omega} = 3\omega^2$

**Sol.17**




$(90 + \alpha) = 180 - 2\theta$

$\tan(90 + \alpha) = \tan(180^\circ - 2\theta)$

$-\cot\alpha = -\tan 2\theta$

$\cot\alpha = \tan 2\theta$

$\tan\alpha = \cot 2\theta$

$\tan\alpha = \frac{1-\tan^2\theta}{2\tan\theta}$

$= \frac{1-1/3}{2x+1/\sqrt{3}} = + \frac{1}{\sqrt{3}} = \alpha = 30^\circ$

$\tan\alpha = \theta$

complex number $= 3\,e^{\circ}\,\frac{2\pi}{3}$

$= \frac{-3}{2} + \frac{3\sqrt{3}}{2}\,i$

**Sol.18** $z_1 + z_2 + z_3 = A,\ z_1 + z_2\omega + z_3\omega^2 = B,$

$z_1 + z_2\omega^2 + z_3\omega = c$

**(a)** adding (1), (2) and (3)

$3z_1 + z_2\,(1 + \omega + \omega^2) + z_3\,(1 + \omega + \omega^2) = A + B + C$

$\Rightarrow z_1 = \frac{A+B+C}{3}$

$(1) + \omega^2(2) + \omega(3)$

$z_1\,(1 + \omega + \omega^2) + 3z_2 + z_3\,(1 + \omega + \omega^2)$

$= A + B\omega^2 + c\omega \qquad \Rightarrow z_2 = \frac{A+B\omega^2+C\omega}{3}$

Similarly $z_3 = \frac{A+B\omega+C\omega^2}{3}$

**(b)** $|A|^2 + |B|^2 + |C|^2 = 3(|z_1|^2 + |z_2|^2 + |z_3|^2)$

$= |z_1|^2 + |z_2|^2 + |z_3|^2 + z_2\bar{z}_1 + z_3\bar{z}_1 + z_1\bar{z}_2 + z_3\bar{z}_2 + z_1\bar{z}_3 + z_2\bar{z}_3$

Similarly for $|B|^2$ & $|C|^2$ and add then

$|A|^2 + |B|^2 + |C|^2 = 3\,(|z_1|^2 + |z_2|^2 + |z_3|^2)$

as $(1 + \omega + \omega^2 = 0)$

**(c)** multiple $z_1$, $z_2$ & $z_3$ from part (a)

$A^3 + B^3 + C^3 - 3ABC = 27\,z_1z_2z_3$

**Sol.19** $z^2 + (P + ip')z + q + iq' = 0$

**(a)** If the euqation has one real root

put $z = x$

$x^2 + (P + ip')x + q + iq' = 0$

$x^2 + px + q + i(p'x + q') = 0$

$x^2 + px + q = 0 \quad p'x + q' = 0$

$x^2 + px + q = 0 \quad x = -\frac{q'}{p'}$

$\left(\frac{q'}{p'}\right) - \frac{pq'}{p'} + q = 0$

$q'^2 - pp'q' + qp'^2 = 0$ H.P.

**(b)** Let roots be $\alpha$ & $\beta$

$\alpha + \beta = -(P + iP')$

$\alpha\beta = q + iq' \qquad \therefore \alpha = \beta$

$2\alpha = -(\pi + ip')$

$\alpha^2 = (q + iq')$

$4\alpha^2 = p^2 - p'^2 + 2ipp'$

$4(q + iq') = p^2 - p'^2 + 2ipp'$

$4q = p^2 - p'2$ & $2q' = pp'$

**Sol.20 (a)** $\left(\frac{1+2i}{2+i}\right)^2 = \left(\frac{(1+2i)(2-i)}{5}\right)^2$

$= \frac{1}{25}[2 - i + 4i + 2]^2 = \frac{1}{25}[4 + 3i]^2$

$= \frac{1}{25}[16 - 9 + 24i]$

$\frac{1}{25}[7 + 24i]$

**(b)** $-(i(9 + 6i)(2 - i)^{-1}$

$= -\left(\frac{9i-6}{2-i}\right) = \frac{6-9i}{2-i}$

$= \frac{(6-9i)(2+i)}{5} = \frac{12+6i-18i+9}{5} = \frac{21-12i}{5}$

**(c)** $\left(\frac{4i^3-i}{2i+1}\right)^2 = \left(\frac{-4i-i}{2i+1}\right)^2 = \left(\frac{-5i(2i-1)}{5}\right)^2$

$= (-i(2i - 1))^2 = 3 + 4i$

**(d)** $\frac{3+2i}{2-5i} + \frac{3-2i}{2+5i}$

$= \frac{(3+2i)}{2-5i} + \frac{3-2i}{2+5i}$

$= \frac{(3+2i)(2+5i)}{29} + \frac{(3-2i)(2-5i)}{29}$

$= \frac{1}{29}[6 + 15i + 4i - 10 + 6 - 15i - 4i - 10]$

$= \frac{-8}{29} + 0i$

**(e)** $\frac{(2+i)^2}{2-i} - \frac{(2-i)^2}{2+i}$

$= \frac{1}{5}[(2 + i)^2 - (2 - i)^3]$

$= \frac{1}{5}[8 - i + 6i(2 + i) - (8 + i - 6i(2 - i)]$

$= \frac{1}{5}[8 - i + 12i - 6 - 8 - i + 12i + 6]$

$= \frac{22}{5}i$

**Sol.21 (a)** $(x + 2y) + i(2x - 3y) = 5 - 4i$

$x + 2y = 5$

$2x - 3y = -4$

By solving $x = 1; y = 2$

**(b)** $(x + iy) + (7 - 5i) = 9 + 4i$

$x + 7 + i(y - 5) = 9 + 4i$

$x + 7 = 9 \Rightarrow x = 2$

$y - 5 = 4 \Rightarrow y = 9$

**(c)** $x^2 - y^2 - i(2x + y) = 2i$

$x^2 + y^2 = 0 \Rightarrow y = \pm x$

$y = x \Rightarrow x = -\frac{2}{3} \Rightarrow y = -\frac{2}{3}$

$y = -x \Rightarrow x = -2 \Rightarrow y = -2$

point $\left(-\frac{2}{3}, -\frac{2}{3}\right)$ $(-2, -2)$

**(d)** $(2 + 3i)x^2 - (3 - 2i)y = 2x - 3y + 5i$

$(2x^2 - 3y) + i(3x^2 + 2y) = 2x - 3y + 5i$

$2x^2 - 3y = 2x - 3y$ & $3x^2 + 2y = 5$

$2x^2 = 2x \Rightarrow x = 0, 1$

$x = 0 \Rightarrow y = \frac{5}{2}$ points $\left(0, \frac{5}{2}\right)$, $(1, 1)$

$x = 1 \quad y = 1$

**(e)** $4x^2 = 3xy + (2xy - 3x^2)i = 4y^2 - \left(\frac{x^2}{2}\right) + (3xy - 2y^2)i$

$4x^2 + 3xy = 4y^2 - \frac{x^2}{2}$ & $2xy - 3x^2 = 3xy - 2y^2$

By solvig $x = k$, $y = \frac{3k}{2}$, $k \in R$

**Sol.22** $|a_1 z^3 + a_2z^2 + a_3z + a_4| = 3$

$3 \le |a_1||z|^3 + |a_2||z|^2 + |a_3||z| + |a_4|$

$3 < |z|^3 + |z|^2 + |z| + 1$

$3 < \frac{1-|z|^4}{1-|z|} \qquad 1 - |z| > 0$

$1 - |z|^4 > 3 - 3|z|$ if $|z| < \frac{2}{3}$

$1 - |z|^4 > 3 - 3|z|$

$3|z| > 2 + |z|^4$

$|z| > \frac{2}{3} + \frac{1}{3}|z|^4$

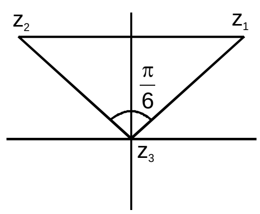
here $|z| > \frac{2}{3}$

**Sol.23** $z_1 = a + i$; $z_2 = 1 + bi$ & $z_3 = 0$

$z_2 = z_1 e^{i\pi/6}$

$1 + bi = (a + i)\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

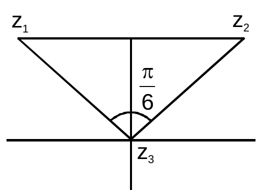
$1 = \frac{a}{2} - \frac{\sqrt{3}}{2}$ ....(1)




$b = \frac{\sqrt{3}}{2}a + \frac{1}{2}$ ....(2)

$a = 2 + \sqrt{3}$ Not possible

$z_1 = z_2\, e^{i\pi/6}$




$a + i = (1 + bi)\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$a = \frac{1}{2} - \frac{\sqrt{3}}{2}b$ ....(3)

$1 = \frac{\sqrt{3}}{2} + \frac{b}{2}$ ....(4)

From (3) & (4) $a = 2 - \sqrt{3}$

$b = 2 - \sqrt{3}$

**Sol.24** **(a)** $\bar{z} = iz^2$

$\Rightarrow$ $|\bar{z}| = |iz^2|$ $\Rightarrow$ $|z| = |z|^2$

$\Rightarrow$ $r = r^2$

$r = 1$ $\Rightarrow$ $z\bar{z} = 1$

$\bar{z} = 1/2$

$\frac{1}{2} = iz^2$

$z^3 = -i$

$z = (-i)^{1/3}$ $\Rightarrow$ $z = i,\ \pm\frac{\sqrt{3}}{2} - \frac{i}{2}$

**(b)** $|z| = 1$ $|z_1| = |z_2| = |z_3| = .... = |z_n| = 1$

$z_1\bar{z}_1 = 1,\ z_2\bar{z}_2 = 1 \;....\; z_n\bar{z}_n = 1$

$|z_1^{-1} + z_2^{-1} + .... + z_n^{-1}| = \left|\frac{1}{z_1} + \frac{1}{z_2} + .... + \frac{1}{z_n}\right|$

$= |\bar{z}_1 + \bar{z}_2 + .... + \bar{z}_n|$

$= |\overline{z_1 + z_2 + .... + z_n}|$

$= |z_1 + z_2 + .... + z_n| = \text{LHS}$

**Sol.25** $|z - 4| + |z + 4| = 16$

$z_1z_2 < 2a$

$8 < 16$ so locus is ellipse

$2a = 16 \Rightarrow a = 8$

$2ae = 8$

$a^2e^2 = 16$

$e^2 = \frac{1}{4}$

$1 - \frac{b^2}{a^2} = \frac{1}{4} \Rightarrow b^2 = 48$

Equation of ellipse will be $\frac{x^2}{64} + \frac{y^2}{48} = 1$

**Sol.26** **(a)** $(1 + \omega - \omega^2)^3 - (1 - \omega + \omega^2)^3$

$= (-2\omega^2)^3 - (-2\omega)^3$

$= -8 + 8 = 0 = \text{RHS}$

**(b)** $(1 - \omega + \omega^2)^5 + (1 + \omega - \omega^2)^5$

$= (-2\omega)^5 + (-2\omega^2)^5$

$= (-2)^5[\omega^2 + \omega] = 2^5 = 32 = \text{RHS}$

**(c)** $(1 + 5\omega^2 + \omega^4)(1 + 5\omega^4 + \omega^2)(5\omega^2 + \omega + \omega^2)$

$= (1 + \omega + \omega^2 + 4\omega^2)(1 + \omega + \omega^2 + 4\omega^4)$

$(4 + 1 + \omega + \omega^2) = (4\omega^2)(4\omega^4)(4) = 64$

**Sol.27** **(a)** $|z + 1 - 2i| = \sqrt{7}$

It represent a circle with centre (–1, 2) & radius = $\sqrt{7}$

**(b)** $|z - 1|^2 + |z + 1|^2 = 4$

$(z - 1)(\bar{z} - 1) + (z + 1)(\bar{z} + 1) = 4$

$z\bar{z} - z - \bar{z} + 1 + z + \bar{z} + z\bar{z} + 1 = 4$

$2z\bar{z} = 2$

$z\bar{z} = 1$

$x^2 + y^2 = 1$

circle with centre at origin and radius = 1

**(c)** $\left|\frac{z-3}{z+3}\right| = 3$

$|z - 3| = 3|z + 3|$

$|z - 3|^2 = 9\,|z + 3|^2$

$(z - 3)(\bar{z} - 3) = 9(z + 3)(\bar{z} + 3)$

$8z\bar{z} + 30z + 30\bar{z} + 72 = 0$

$8(x^2 + y^2) + 30x + 72 = 0$

$x^2 + y^2 + \frac{15}{2}x + 9 = 0$

centre $= \left(-\frac{15}{4}, 0\right)$ Radius $= \frac{9}{4}$

**(d)** $|z - 3| = |z - 6|$

squaring both side

$|z - 3|^2 = |z - 6|^2$

$(z - 3)(\bar{z} - 3) = (z - 6)(\bar{z} - 6)$

$z\bar{z} - 3z - 3\bar{z} + 9 = z\bar{z} - 6z - 6\bar{z} + 36$

$3z + 3\bar{z} - 27 = 0$

$z + \bar{z} - 9 = 0$

$x + iy + x - iy - y = 0$

$2x - 9 = 0$ a st. line.

**Sol.28 (i)** $6(\cos 310^\circ - i\sin 310^\circ)$

$6[\cos(2\pi - 50^\circ) - i\sin(2\pi - 50^\circ)]$

$6[\cos 50^\circ + i\sin 50^\circ]$

modulus = 6

principal Arg $= \frac{5\pi}{18}$

Arg $= 2kp + \frac{5\pi}{18}$, where $k \in I$

**(ii)** $-2(\cos 30^\circ + i\sin 30^\circ)$

$2\left[-\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right]$

$2\left[\cos\left(\pi + \frac{\pi}{6}\right) + i\sin\left(\pi + \frac{\pi}{6}\right)\right]$

$2\left[\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right]$

Modulus = z

Principal Arg $= \frac{7\pi}{6}$

Arg $= 2m\pi + \frac{7\pi}{6}$ $m \in I$

**(iii)** $z = \frac{z+i}{4i + (1+i)^2}$

$= \frac{2+i}{4i + 1 - 1 + 2i} = \frac{2+i}{6i}$

$= -\frac{1}{6}i(z + i) = -\frac{1}{6}(-1 + 2i)$

Modulus $= \sqrt{\frac{1}{36} + \frac{1}{9}} = \frac{\sqrt{5}}{6}$

Principal Arg $= \tan^{-1}\left(\frac{-\frac{1}{5}}{\frac{1}{6}}\right) = -\tan^{-1} 2$

Arg $z = 2k\pi - \tan^{-1}2$ $K \in I$

**Sol.29** $|1 - z_1\bar{z}_2|^2 - |z_1 - z_2|^2$

$= (1 - z_1\bar{z}_2)(1 - \bar{z}_1 z_2) - (z_1 - z_2)(\bar{z}_1 - \bar{z}_2)$

$= 1 - \bar{z}_1 z_2 - z_1\bar{z}_2 + |z_1|^2|z_2|^2 - |z_1|^2 + z_1\bar{z}_2$

$+ z_2\bar{z}_1 - |z_2|^2$

$= 1 - |z_1|^2 + |z_1|^2|z_2|^2 - |z_2|^2$

$= (1 - |z_1|^2)(1 - |z_2|^2) =$ RHS H.P.

**Sol.30 (i)** $(1 + \omega - \omega^2)^3 - (1 - \omega + \omega^2)^3$

$= (-2\omega^2)^3 - (-2\omega)^3 = -8 + 8 = 0$

**(ii)** $\frac{a + b\omega + c\omega^2}{c + a\theta + b\omega^2} = \frac{\omega^2(a + b\omega + c\omega^2)}{(c\omega^2 + a + b\omega)} = \omega^2$

**(iii)** $(1 - \omega)(1 - \omega^2)(1 - \omega^4)(1 - \omega^8)$

$= (1 - \omega)(1 - \omega^2)(1 - \omega)(1 - \omega^2)$

$= (1 - \omega)(1 - \omega^2)^2$

$= (1 + \omega^2 - 2w)(1 + \omega^4 - 2\omega^2)$

$= (1 + \omega^2 - 2\omega)(1 + \omega - 2\omega^2)$

$= (-3\omega)(-3\omega^2) = 9 =$ RHS

**Sol.31** $z = a + b$; $y = a\omega + b\omega^2$; $z = a\omega^2 + b\omega$

**(i)** $xyz = (a + b)(a\omega + b\omega^2)(a\omega^2 + b\omega)$

$= (a + b)(a^2 + ab\omega^2 + ab\omega + b^2)$

$= (a + b)(a^2 - ab + b^2) = a^3 + b^3$

**(ii)** $x^2 + y^2 + z^2 + (a + b)^2 + (a\omega + b\omega^2)^2$

$+ (a\omega^2 + b\omega)^2$

$= 6ab$

**(iii)** $x^3 + y^3 + z^3 = (a + b)^3 + (a\omega + b\omega^2)^3$

$+ (a\omega^2 + b\omega)^3$

$= (a + b)^3 + (a + b\omega)^3 + (a + b\omega^2)^3$

$= 3a^3 + 3b^3 + 3ab(a + b) + 3ab\omega(a + b\omega)$

$+ 3ab\omega^2(a + b\omega^2)$

$= 3a^3 + 3b^3 = 3(a^3 + b^3)$